

A modified-Lorentz-Transformation based gravity model confirming basic GRT experiments.

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Abstract

Implementing Poincaré's *geometric conventionalism* a scalar Lorentz-covariant gravity model is obtained based on gravitationally modified Lorentz transformations (or GMLT). The modification essentially consists of an appropriate space-time and momentum-energy scaling ("normalization") relative to a nondynamical flat background geometry according to an isotropic, nonsingular gravitational *affecting* function $\Phi(\mathbf{r})$. Elimination of the gravitationally *unaffected* S_0 perspective by local composition of space-time GMLT recovers the local Minkowskian metric and thus preserves the invariance of the locally observed velocity of light. The associated energy-momentum GMLT provides a covariant Hamiltonian description for test particles and photons which, in a static gravitational field configuration, endorses the four 'basic' experiments for testing General Relativity Theory: gravitational *i)* deflection of light, *ii)* precession of perihelia, *iii)* delay of radar echo, *iv)* shift of spectral lines. The model recovers the Lagrangian of the Lorentz-Poincaré gravity model by Torgny Sjödin and integrates elements of the precursor gravitational theories, with spatially Variable Speed of Light (VSL) by Einstein and Abraham, and gravitationally variable mass by Nordström.

Key words: modified Lorentz Transformations, scalar gravitation.

1 Introduction

It is well known that it is possible to treat gravitation in a flat, "unrenormalized" pseudo-Euclidean space-time, *i.e.* the one used in special relativity. Then, because of the universal action of gravity, even the meter sticks and the clocks are affected by gravity. The "renormalized" space-time observed by those modified rods and clocks turns out to be Riemannian. By the standard treatment of the field theory, *i.e.*, assumption of the existence of a potential, Lagrangian with minimal coupling, and variational principle, both the field equations and the equations of motion are derived. The latter ones are in general not the Euler-Lagrange equations but the ones of Infeld-Khalatnikov-Kalman [1] generalized to cope with Finsler spaces by Cavalleri and Spinelli [2]. The agreement with observations depends on the tensor rank of the potential. Actually, a *scalar* theory (*i.e.*, a theory whose potential is a zero-rank tensor) gives no deviation of the light ray, and a perihelion precession $-1/6$ the correct value [3]. A *vector* theory (*i.e.*, a theory whose potential is a first-rank tensor) has to be immediately discarded since in an attractive theory—as gravitation—it leads to self-acceleration, moreover it violates the Weak Equivalence Principle [4]. A *second rank* gravitational theory has been initiated by Fierz [5], was subsequently developed by many authors [6, 7] and shown to be valid even with the most general gauge by Cavalleri and Spinelli [8]. The corresponding "renormalized" space-time is Riemannian and the theory, obtained by an iterative procedure, converges to Einstein's theory. From the above point of view, any new scalar gravitational theory seems to be discarded.

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However, what is proposed here, is a modified scalar theory leading to the same predictions derivable from the PPN approximation [9, 10], i.e. to the expansion of Einstein’s theory into flat space but truncated after the second order in κ/r .

However this model differs from Einstein’s theory; e.g. it does not lead to “black holes”, in the sense that the speed of light decreases with r but vanishes only for $r \rightarrow 0$. The divergence for finite r —considered as a failure by Einstein himself— is not present in it. Moreover there is some hint from astronomy, that the γ -ray bursts could be explained by the collision of a binary neutron star system, but each with a mass $> 5M_{Sol}$ as required for the observed energy emission. *I.e.* well beyond the limit of collapse to a black hole according to GRT ⁽¹⁾. It is therefore worthwhile to develop in particular, a nonsingular, modified scalar theory whose predictions are in agreement with present gravitational experiments.

The ‘modified’ scalar gravity model is inspired by the principle of gravitation as a physical affecting of space and time measurement rather than intrinsic geometric modification [11]. The effectiveness of the model relies on Poincaré’s conventionalism of geometry. Poincaré’s concern was the complementing nature of geometry and physics (optics), leading to a theoretical indetermination which precludes the extraction of the unique fundamental geometry of nature. Poincaré’s well known argument states that a physical theory is a set of laws which are a combined result of geometry (‘G’) and physical rules (‘P’), and, this inextricable combination (‘G+P’) leaves open to convention the nature of the intrinsic basic geometry [12].

Various attempts to model gravitation in the Lorentz-Poincaré sense —i.e. with Euclidean space and time and physical effects like ‘rod contraction and clock dilation’— have been made (cf. references in [13, 14], and Thirring’s “normalization” [15]), most often invoking spatially variable speed of light (compare to recent VSL-theories for epochal variation of universal constants in cosmology, e.g. in the work of J.W. Moffat [16], and A. Albrecht and J. Magueijo [17]). Historically the main precursor theories of gravitation were conceived using a gravitationally dependent velocity of light (e.g., Einstein [18] and Abraham [19]) or mass (Nordström [20]). The ineffectiveness of the approach —the manifest absence of Lorentz symmetry— was subsequently recognized (see e.g. Norton’s overview [21]).

The present model uses a spatially variable speed of light but establishes the local Lorentz symmetry due the introduction of scaling functions on measured space and time intervals. The model recovers the particle Lagrangian of Torgny Sjödin’s Lorentz-Poincaré type gravity model [22-24], includes the VSL-property of the modified Lorentz transformations written by Abraham [19] in continuation of related developments by Einstein [18], and aspects of mass variability according Nordström’s scalar gravitation theory [20].

The gravity model —in the present paper only developed for test particles in a static spherically symmetric configuration— does endorse the four ‘basic’ GRT experiments to required order in the Schwarzschild radius κ while retaining a Lorentz-Poincaré interpretation.

In the following Section (2) we develop the concept of gravitationally modified Lorentz transformation (GMLT). Modifications, or generalizations, of the Lorentz transformations —introduced previously for different purposes in the literature, e.g. [25]— break Lorentz covariance in some limit, e.g. at small energy scale. In these limits the modified transformations lack the Lorentz symmetry but still relate quantities as observed by different observers. In the present model the proposed modifications are due to gravitation and aim at an adequate description of relativistic gravitational effects. The particle and photon Hamiltonian, derived by GMLT, are rendered by applying the Newtonian fit. These expressions allow one to compare the present scalar gravitation model with the *deficient* classical scalar and vectorial models, as outlined by Cavalleri and Spinelli [26].

Section (3) presents some applications and consequences of these transformations, in particular calculations of the ‘basic’ GRT experiments and kinematic modification of gravitational accel-

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eration. The successful description of the ‘basic’ experiments are evidently not sufficient for considering the present model as yet a full alternative metrization of GRT. We refer to recent work for some further development of the model concerning dynamic sources [9] and extended test masses [10] in line with GRT.

We remark the recent scientific literature accounts of numerous—in aspects—related approaches to our model, e.g. in the sense of either: VSL-models [27, 17, 16], polarizable vacuum models [28], scalar field gravitation models [14, 29], nonsingular ether gravitation models [30] and, field conceptions of gravitation [31-33].

2 Gravitationally modified Lorentz transformations

Following the geometric conventionalist approach, we introduce a gravitational affecting of observed space and time intervals (‘rod intervals’ and ‘clock intervals’) in a gravitational field, *i.e.* the physical *congruence standard*. We let observed space and time intervals, and the speed of light be heterogeneously and isotropically affected. The gravitational affecting at position \mathbf{r} is given by the function $\Phi(\mathbf{r})$, $0 \leq \Phi(\mathbf{r}) \leq 1$, as the ratio of a space interval observed by the gravitationally unaffected S_0 with *coordinate* geometry, and the same infinitesimal interval as observed by gravitationally affected observer S' with *natural* geometry, both at \mathbf{r} .

$$d\mathbf{x} = d\mathbf{x}'\Phi(\mathbf{r}) \quad , \quad dt = \frac{dt'}{\Phi(\mathbf{r})}, \quad (1)$$

where the inverse scaling effect is set on the relation of affected and unaffected infinitesimal time intervals. Within the scope of the present model there is no need to attribute to this time ratio a separate affecting function (compare [23]). The invariance of the *locally observed* velocity of light is secured by adequately balancing the variability of speed of light and gravitational affecting of space and time measurement:

$$c(\mathbf{r}) = c'\Phi(\mathbf{r})^2. \quad (2)$$

The imposed ratio of velocities of light $c(\mathbf{r})$ relative to gravitationally unaffected S_0 , and c' as observed by affected observer S' , is consistent with affected space and time ratios. The locally observed velocity of light ‘in vacuum’ remains the usual universal constant c' . We emphasize that relative to the gravitationally unaffected observer S_0 we have posited an explicitly variable speed of light, and explicit ‘observed’ time dilation and observed space interval contraction. All observed variables, $\{d\mathbf{x}', dt', c'\}$, are primed and refer to the affected observer S' , while all variables $\{d\mathbf{x}, dt, c\}$ without prime refer to the ‘unphysical’ perspective of the unaffected S_0 . If the affecting function $\Phi(\mathbf{r})$ is known, the latter variables can immediately be related to those of S' by calculation. Actually expressions should refer to the physical S' perspective. Still, comparison with GRT’s results is done most often in the coordinate perspective of S_0 , which does not necessitate a protocol for measurement of extensive length. Finally we note that, the general case of relations Eq. (1) with $\Phi = \Phi(\mathbf{r}, t)$ are equally valid with respect to the gravitational affecting principle. However, the static field configuration is sufficient for the scope of the present paper.

Subsequently we introduce the effect of kinematics on observed space and time intervals in a given static field $\Phi(\mathbf{r})$.

2.1 Space and Time GMLT

In general a purely gravitational affecting Eq. (1) and *kinematic* affecting are required in the observer transformations. In the Lorentz-Poincaré interpretation the latter consists of, *i)* physical longitudinal Lorentz contraction of a ‘material rod’ by a factor $\gamma(v)^{-1}$, *ii)* physical Lorentz

time dilation of a moving ‘clock’ by a factor $\gamma(v)$, both with velocity \mathbf{v} relative to the field Φ . The composition of both effects —gravitational and kinematic— and *standard synchronization* or *Einstein synchronization* [34-37] into an observer transformation between momentary locally coincident observers S' and S_0 , leads directly to the gravitationally modified Lorentz transformation, the GMLT of S_0 to S' :

$$d\mathbf{x}' = ((d\mathbf{x}_{\parallel} - \mathbf{u}dt)\gamma(u) + d\mathbf{x}_{\perp}) \frac{1}{\Phi(\mathbf{r})}, \quad (3)$$

$$dt' = \left(dt - \frac{\mathbf{u} \cdot d\mathbf{x}}{c(\mathbf{r})^2}\right) \gamma(u) \Phi(\mathbf{r}), \quad (4)$$

and the inverse GMLT S' to S_0 :

$$d\mathbf{x} = ((d\mathbf{x}'_{\parallel} - \mathbf{u}' dt')\gamma(u') + d\mathbf{x}'_{\perp}) \Phi(\mathbf{r}), \quad (5)$$

$$dt = \left(dt' - \frac{\mathbf{u}' \cdot d\mathbf{x}'}{c'^2}\right) \frac{\gamma(u')}{\Phi(\mathbf{r})}, \quad (6)$$

with $c(\mathbf{r})$ as in Eq. (2), and $\gamma(u)$ and \mathbf{u} satisfying:

$$\mathbf{u} = -\mathbf{u}' \Phi(r)^2, \quad (7)$$

$$\gamma(u') = \left(1 - \left(\frac{u'}{c'}\right)^2\right)^{-1/2} = \left(1 - \left(\frac{u}{c(\mathbf{r})}\right)^2\right)^{-1/2}. \quad (8)$$

From which the usual velocity relation, angle conversion, and γ relations are obtained, S_0 to S' :

$$\mathbf{v}' = \frac{\mathbf{v}_{\parallel} - \mathbf{u} + \gamma(u)^{-1} \mathbf{v}_{\perp}}{\left(1 - \mathbf{u} \cdot \mathbf{v} c(\mathbf{r})^{-2}\right) \Phi(\mathbf{r})^2}, \quad \mathbf{v} = \frac{d\mathbf{x}}{dt}, \quad \mathbf{v}' = \frac{d\mathbf{x}'}{dt'}, \quad (9)$$

$$\tan \theta' = -\frac{\tan \theta}{\gamma(u) (1 - uv_{\parallel}^{-1})}, \quad \theta = \angle(\mathbf{u}, d\mathbf{x}), \quad \theta' = \angle(\mathbf{u}', d\mathbf{x}'), \quad (10)$$

$$\gamma(v') = \gamma(v) \gamma(u) (1 - \mathbf{u} \cdot \mathbf{v} c(r)^{-2}). \quad (11)$$

This first type of GMLT Eqs. (5, 6) or eqs. (3, 4) relate affected S' and unaffected S_0 observers, and therefore need not constitute a Lorentz group. These transformation are in fact related to the transformations used in early gravitation models by Abraham [19] which elaborated the concept of ‘light velocity as gravitational potential’ introduced, and subsequently rejected on grounds of relativistic incompatibility, by Einstein [18]. In their approach the modified transformations related physical, thus affected, observers. In the present model Lorentz symmetry is maintained between *affected* locally coincident observers, *i.e.* $\Phi_1 = \Phi_2$, for then the composition of two GMLT’s S'_1 to S_0 eqs. (5, 6) and S_0 to S'_2 eqs. (3, 4), by elimination of their mutual reference to S_0 , leads to a regular Lorentz transformation.

In the general case the space time relations between affected observers lead to the introduction of the second type of space-time modified Lorentz transformations. These GMLT relate two distinctly, gravitationally and kinematically, affected observers S'_1 and S'_2 , $\Phi_1 \neq \Phi_2$, $\mathbf{u}_1 \not\parallel \mathbf{u}_2$. These are again obtained by elimination of the S_0 perspective from a composition of two GMLT’s eqs. (5, 6) and eqs. (3, 4). In the Appendix these GMLT expressions are given taking into account that no kinematical reference should be made relative to the “preferred” frame of S_0 . The presence of scaling factors Φ_1/Φ_2 and distinct speeds of light $c_1 = c(\mathbf{r}_1)$ and $c_2 = c(\mathbf{r}_2)$ in the resulting transformation S'_1 to S'_2 prevent one to obtain the Lorentz symmetry (e.g. [38], sec. 2.17). However, in a gravitational field the Lorentz symmetry needs only be regained

in the local limit $\Phi_1 \rightarrow \Phi_2$, which requirement is satisfied (Appendix eqs. (82, 83)). In fact all expressions —relative frame velocities, kinematical contraction factors, Thomas angle— are verified to smoothly recover the Special Relativistic expressions in the local limit.

Standard literature on GRT describes celestial mechanical problems in the coordinate space perspective S_0 . Comparison of present results with GRT will therefore not require a GMLT between affected observers. Even, most problems are solved more conveniently relative to S_0 , and if necessary results can be transformed into any affected perspective S' afterwards.

2.2 Energy and Momentum GMLT

The presence of the scaling function and spatial variability of the speed of light in the space-time modified Lorentz transformations S'_0 to S' eqs. (3, 4) impedes the immediate transcription into momentum-energy modified Lorentz transformations. In order to derive the momentum-energy GMLT, we multiply the sides of the space-time GMLT Eq. (3) and Eq. (4) by

$$m'_0 \gamma(v') \frac{1}{dt'} = m'_0 \frac{\gamma(v)}{\Phi(\mathbf{r})} \frac{1}{dt}, \quad (12)$$

where m'_0 is the rest mass of a test body as attributed by an affected locally coincident observer S' . With an additional factor c'^2 Eq. (4) and consistent reordering into the Lorentz transformation *form*, we obtain:

$$\begin{aligned} m'_0 \gamma(v') \mathbf{v}' &= \left(\left(m'_0 \frac{\gamma(v)}{\Phi(\mathbf{r})^{3+\delta}} \mathbf{v}_{\parallel} - m'_0 \frac{\gamma(v)}{\Phi(\mathbf{r})^{3+\delta}} c(\mathbf{r})^2 \frac{\mathbf{u}}{c(\mathbf{r})^2} \right) \gamma_u + m'_0 \frac{\gamma(v)}{\Phi(\mathbf{r})^{3+\delta}} \mathbf{v}_{\perp} \right) \Phi(\mathbf{r})^{1+\delta} \\ m'_0 \gamma(v') c'^2 &= \left(m'_0 \frac{\gamma(v)}{\Phi(\mathbf{r})^{3+\delta}} c(\mathbf{r})^2 - \mathbf{u} \cdot m'_0 \frac{\gamma(v)}{\Phi(\mathbf{r})^{3+\delta}} \mathbf{v} \right) \gamma_u \frac{1}{\Phi(\mathbf{r})^{1-\delta}}, \end{aligned} \quad (14)$$

where δ is an as yet to be fixed numerical parameter. Equations (13) and (14) lead to a consistent interpretation of the test body's relative energy and momentum by an affected observer S' , locally coincident with the test body and attributing to it a relative velocity \mathbf{v}' , according:

$$\mathbf{p}' \equiv m'_0 \gamma(v') \mathbf{v}', \quad E' \equiv m'_0 \gamma(v') c'^2, \quad (15)$$

and the corresponding momentum and energy as attributed by S_0 :

$$\mathbf{p} \equiv m(\mathbf{r}, v) \mathbf{v}, \quad E \equiv m(\mathbf{r}, v) c(\mathbf{r})^2, \quad (16)$$

with location and velocity dependent mass $m(\mathbf{r}, v)$ in the S_0 perspective:

$$m(\mathbf{r}, v) \equiv m'_0 \frac{\gamma(v)}{\Phi(\mathbf{r})^{3+\delta}}. \quad (17)$$

The momentum-energy GMLT eqs. (13, 14) are *a priori* not unique as the powers of the factorized affecting functions depend on the free numerical parameter δ . However, by fitting the model to the Newtonian limit and experiments, this adaptable parameter is fixed at

$$\delta = 0. \quad (18)$$

We will therefore adopt GMLT eqs. (13, 14) immediately with Eq. (18) and only mention the effect of the adaptable parameter δ again when appropriate.

The location dependence of mass Eq. (17) relative to S_0 , considering Eq. (18), corresponds qualitatively to Mach's principle of mass induction: mass increases when approaching the gravitational source ($\Phi \rightarrow \Phi_{min.}$), but remains finite on asymptotic separation $m_{\infty} = m'_0$ ($\Phi_{\infty} = 1$).

The fundamental expression, relative to S_0 , for the energy $E = E(\mathbf{r}, \mathbf{p})$ of a test body with rest mass m_0 in a gravitational field Φ is obtained by elimination of the velocity \mathbf{v} in Eq. (16):

$$E^2 - c(\mathbf{r})^2 p^2 = m_0(\mathbf{r})^2 c^4, \quad (19)$$

which is the formal equivalent of the energy relation for a free mass in SRT. By extension, in classical light approximation $m_0 \equiv 0$, the energy of a photon in a gravitational field is given by:

$$E = pc(\mathbf{r}). \quad (20)$$

With the definitions of momentum Eq. (16 a), energy Eq. (16 b), and mass Eq. (17), in the unaffected perspective of S_0 the momentum-energy GMLT S_0 to S' becomes:

$$\mathbf{p}' = \left(\left(\mathbf{p}_{\parallel} - \frac{E}{c(\mathbf{r})^2} \mathbf{u} \right) \gamma(u) + \mathbf{p}_{\perp} \right) \Phi(\mathbf{r}), \quad (21)$$

$$E' = (E - \mathbf{p} \cdot \mathbf{u}) \frac{\gamma(u)}{\Phi(\mathbf{r})}, \quad (22)$$

while the inverse transformation S' to S_0 is given by:

$$\mathbf{p} = \left(\left(\mathbf{p}'_{\parallel} - \frac{E'}{c'^2} \mathbf{u}' \right) \gamma(u') + \mathbf{p}'_{\perp} \right) \frac{1}{\Phi(\mathbf{r})}, \quad (23)$$

$$E = (E' - \mathbf{p}' \cdot \mathbf{u}') \gamma(u') \Phi(\mathbf{r}). \quad (24)$$

Similarly as in the space-time transformations we do not recover Lorentz symmetry between affected and unaffected observers. We only recover the SRT Lorentz transformations for energy-momentum for locally coincident or similarly affected observers. In the (\mathbf{p}, E) -GMLT the departure from the Lorentzian symmetry in the noncoincident configuration of affected observers will be effective in causing the required gravitational dynamics.

2.3 GMLT metric and invariants

The (\mathbf{p}, E) -GMLT and (\mathbf{x}, t) -GMLT differ only in the overall Φ factor which is relatively counterposed. The relatively inverse gravitational influence in space-time GMLT and momentum-energy GMLT excludes a purely ‘metric’ interpretation of this gravitation model. While the space-time GMLT leads to the expression of invariant line element ds'^2 and metric

$$ds'^2 = c'^2 dt'^2 - d\mathbf{x}'^2 = \Phi(r)^{-2} \left(c(\mathbf{r})^2 dt^2 - d\mathbf{x}^2 \right), g_{\mu\nu} = \{\Phi^2, -\Phi^2 \mathbf{1}\}, \quad (25)$$

the momentum-energy GMLT Eq. (21, 22) leads to a distinct ‘momentum-energy metric’ as well.

$$m_0'^2 c'^4 = E'^2 - p'^2 c'^2 = \Phi(r)^{-2} \left(E^2 - c(\mathbf{r})^2 p^2 \right), g_{\mu\nu} = \{\Phi^{-2}, -\Phi^2 \mathbf{1}\}. \quad (26)$$

In this scalar model physical quantities are related to adjusted GMLT’s, depending on their dimensional units. Consequently, all natural ‘constants’ can not *a priori* be considered unaffected by gravitation and could be covariant with a GMLT. E.g., we observe this feature in the velocity of light c , but on the contrary not in Planck’s constant h or the electromagnetic fine structure constant α . The invariance of α is trivial due to its dimensionless nature ⁽²⁾, while the invariance

²The fine structure constant, using SI units, is given by $\alpha = e^2/(4\pi\epsilon_0\hbar c)$. Since $c = (\epsilon_0\mu_0)^{-1/2}$, the invariance of α is due to permeability and permittivity transforming identically over GMLT. The respective units are $[\mu] = Js^2/(C^2m)$ and $[\epsilon] = C^2/(Jm)$. From the GMLT’s we see $J \rightarrow J'\Phi$, $m \rightarrow m'\Phi$ and $s \rightarrow s'\Phi^{-1}$, then $[\mu]/[\mu'] = [\epsilon]/[\epsilon']$. From the invariance of α no inference can be made about the possible gravitational affecting of electric charge $C \rightarrow C'\Phi^n$. On the other hand, when Gauss units are used, the fine structure constant is given by $\alpha = e^2/(\hbar c)$. Then the invariance of α and \hbar lead to $e^2/c = e'^2/c'$, and from Eq. (2) now follows $e' = e\Phi^{-1}$. *I.e.* gravitational affecting of electric charge when using statcoulomb units.

of Planck's constant follows from the combination of (\mathbf{p}, E) -GMLT and (\mathbf{x}, t) -GMLT covarying quantities. The simplification is due to the contraposition of the Φ factor in (\mathbf{x}, t) -GMLT eqs. (3, 4) and (\mathbf{p}, E) -GMLT eqs. (21, 22):

$$\mathbf{p}' \cdot d\mathbf{x}' - E' dt' = \mathbf{p} \cdot d\mathbf{x} - E dt. \quad (27)$$

Taking into account the covariance of the Einstein-Compton relations for corpuscular light relative to the GMLT S_0 and S' :

$$E = h\nu, \quad p = \frac{h}{\lambda} \quad \text{and} \quad E' = h'\nu', \quad p' = \frac{h'}{\lambda'}, \quad (28)$$

we have, with $\mathbf{k} = \boldsymbol{\lambda}/\lambda^2$, $\mathbf{k}' = \boldsymbol{\lambda}'/\lambda'^2$

$$h'(\mathbf{k}' \cdot d\mathbf{x}' - \nu' dt') = h(\mathbf{k} \cdot d\mathbf{x} - \nu dt). \quad (29)$$

Then, given the trivial invariance of a dimensionless phase, the GMLT-invariance of Planck's constant is obtained:

$$h = h'. \quad (30)$$

The straightforward but tacit conditions for the validity of this result are: *i*) $\Delta\Phi \rightarrow 0$ when $\Delta x \rightarrow \lambda$ for Eq. (29), and foremostly *ii*) the adaptable parameter δ as required by its fixing Eq. (18). In subsection Eq. (3.4) we apply the invariance of Planck's constant Eq. (30) in solving the problem of the spectral shift of light in gravitational fields, requiring and therefore endorsing the choice Eq. (18) for δ .

2.4 Newtonian fit and static field equation

The identification of Φ needs to be done in order to solve mechanical problems in practice. This will be done by comparing expressions of the energy change when a mass is lowered in a gravitational field according two affected observers: the first observer, S'_w , is at rest relative to the background field Φ , and the second observer, S'_e , is the eigen observer of the lowered test mass. In order not to have kinematical contributions we consider the test mass to be at rest in the field at the start and end of the lowering. This will allow the static observer S'_w to identify the energy shift of the mass as a pure Newtonian potential energy shift. The eigen observer S'_e will invariably attribute $\mathbf{p}'_e = 0$ and $E'_e = m'_0 c'^2$ both at the start and end of the lowering of the mass. In this static configuration the energy GMLT between S'_e and S'_w is simply:

$$E'_w \Phi_w = E'_e \Phi_e. \quad (31)$$

The lowering of the test body at S'_e leads to a change in static Newtonian potential energy $E'_w = U'(r')$ relative to S'_w , while for S'_e the energy remains $E'_e = m'_0 c'^2$, thus:

$$\Phi_w dE'_w = E'_e d\Phi_e. \quad (32)$$

The lowering of the mass in the end makes S'_e and S'_w locally coincident, *i.e.* $\Phi_w = \Phi_e$, then:

$$\frac{d\Phi_w}{\Phi_w} = \frac{1}{m'_0 c'^2} d \left(-G' m'_0 \int_S \frac{\rho(\mathbf{r}'')}{|\mathbf{r}' - \mathbf{r}''|} d^3 r'' \right). \quad (33)$$

This directly solves to the expression:

$$\Phi_w(\mathbf{r}') = \exp \left[-\frac{G'}{c'^2} \int_S \frac{\rho(\mathbf{r}'')}{|\mathbf{r}' - \mathbf{r}''|} d^3 r'' \right]. \quad (34)$$

Then Φ_w , specifically outside a spherically symmetric source of mass M' , is given by:

$$\Phi_w(r') = \exp\left[-\frac{\kappa'}{r'}\right], \quad \kappa' \equiv -\frac{G'M'}{c'^2}, \quad (35)$$

where κ' is formally equal to half the (critical) Schwarzschild radius. A priori, no singular features are expected in the affecting function Φ , conform the gravitational scaling of space and time intervals by Φ , Eq. (1). Evidently the accuracy of the Newtonian fit does not allow a physical interpretation of this closed exponential form, of which the valid expansion order in κ should be verified in comparison with GRT predictions. When treating the GRT problems in the next Section we will require the expression $\Phi(\mathbf{r})$ from the perspective of S_0 instead of S'_w . The exact transcription of $\Phi_w(\mathbf{r}')$, Eq. (34), into the S_0 perspective requires a conventionally defined measurement protocol for extensive length.

If on the other hand the unaffected observer S_0 attributes due the lowering of the test mass a *scaled* Newtonian energy difference equal to $dE = \Phi dU_N(r)$, then a field equation in S_0 perspective can be obtained. This energy shift attribution merely implies that for S_0 the Newtonian energy difference is affected identically as rest mass is by the gravitational field. In that case the Newtonian fitting procedure, with $dE = m'_0 c'^2 d\Phi$, from the perspective of S_0 leads to:

$$m'_0 c'^2 d\Phi = \Phi dU_N(r), \quad (36)$$

and the field equation in S_0 perspective follows:

$$\Delta\Phi = \frac{4\pi G'}{c'^2} \rho(\mathbf{r})\Phi + \frac{(\nabla\Phi)^2}{\Phi}, \quad \Phi = \exp\left[-\frac{G'}{c'^2} \int_S \frac{\rho(\mathbf{r}^*)}{|\mathbf{r} - \mathbf{r}^*|} d^3r^*\right]. \quad (37)$$

For a spherically symmetric source of radius R and mass M this leads to:

$$\Phi(\mathbf{r}) = \exp\left[-\frac{\kappa}{r}\right], \quad r > R, \quad \kappa \equiv -\frac{G'M}{c'^2}. \quad (38)$$

The affecting function $\Phi(\mathbf{r})$ thus consistently satisfies a static gravitational field equation⁽³⁾ with source terms due material density $\rho(\mathbf{r})$ and the gravitational field energy density $(\nabla\Phi)^2/\Phi$.

Expression (38) of $\Phi(\mathbf{r})$, as a scaling function, satisfies the conditions of monotonous increase $\partial_r\Phi > 0$ ($r \neq 0$), boundedness $0 \leq \Phi \leq 1$, and unaffected limit $\Phi_\infty = 1$. While the closed exponential form $\Phi(\mathbf{r})$ is now sustained by the field equation, its validity is still subject to comparison with endorsed GRT predictions. In view of the closed form Eq. (38) —appropriate for gravitational scaling— any critical phenomena at $r = \kappa$ appears only due to truncation of order in the coupling parameter G .

In the preceding development we have conservatively taken the mass density ρ as the primary source of the gravitational field in the Laplace equation. In further development of the model, and if required by theoretical extension, supplementary source terms like stress tensor $T_{\mu\mu}$ or electromagnetic energy density should be considered. Finally we mention once more the effect of the adaptable parameter δ when different from its fixed value Eq. (18): the procedure of the Newtonian fit with parameter δ leads to $\Phi(\mathbf{r}) = \exp[-\kappa/(1-\delta)1/r]$. Then the scaling boundary conditions require $\delta < 1$. We notice the adaptable parameter changes the effective Schwarzschild radius, this feature is not required in obtaining correct predictions of gravitational mechanics.

³Based on invariance requirements, a similar equation for a point source was derived by Torgny Sjödin (private communication). Equation (37) resembles the static field equation proposed by Einstein in his precursor VSL-gravitation theory ([18], **38**, 456, Eq. (3b)): $c\Delta c - \frac{1}{2}(\nabla c)^2 = kc^2\rho$, with c the velocity of light, k the universal gravitational constant, and ρ mass density (see also [21]).

2.5 Effective tensorial rank of interaction

With the premisses for the scalar gravitational model established, we can now discuss the generally accepted ‘no-go’ assessment of such models as mentioned in the introductory section, e.g. by Cavalleri and Spinelli [26]. In order to compare the modified scalar model to classical field models we compare the Lagrangians in the ‘minimal coupling’ description ([26], Eq.(2)). Applying the Legendre transformation on Eqs. (19, 20), gives the Lagrangian expressions:

$$L_{part} \equiv \mathbf{p} \cdot \mathbf{v} - H_{part.} = -m'_0 \frac{\Phi}{\gamma(v)} \quad , \quad L_{phot.} \equiv \mathbf{p} \cdot \mathbf{v}_c - H_{phot.} = 0. \quad (39)$$

We do obtain the correct limit relation $v \rightarrow c$ between particles and photons. But a perturbation development of the photon Lagrangian on v_c is not possible ($v_c/c = 1$). For particles we obtain, using $\Phi = \exp[\varphi]$, Eq. (38), the approximation:

$$L_{part} = -m'_0 c'^2 \left(1 + \varphi + \frac{1}{2} \varphi^2\right) \left(1 - \frac{1}{2} \frac{v^2}{c'^2} (1 - 4\varphi + 8\varphi^2)\right) + O(\kappa^2, v^4/c'^4). \quad (40)$$

The components of the interaction and free Lagrangian are easily identified. We obtain, with $v^0 \equiv 1$, $L_{part} = L_0 + L_{int.}$ and minding the hybrid form $\gamma(v/c')$, to first order in the coupling parameter (which is sufficient for the present purpose);

$$L_0 = -\frac{m'_0 c'^2}{\gamma(v/c')}, \quad (41)$$

$$L_{int.} = -m'_0 c'^2 \psi_{\alpha\beta} \frac{v^\alpha v^\beta}{c'^2} + O(\kappa^2, \kappa v^4/c'^4), \quad \psi_{\alpha\beta} = \left\{1, \frac{3}{2} \delta_{ij}\right\} \varphi, \quad (42)$$

which is the first-order part of the particle Lagrangian in GRT ([39], Eq. 9.2.3). In effect the minimal coupling description of the particle Lagrangian identifies the scalar model as rank-2 tensor model, with static source potential $\psi_{\alpha\beta}$. This behaviour comes about through $c(r)$ in the kinematical γ -factor. The ratio v^2/c^2 systematically gives the appearance of higher rank coupling; $v^2/c^2 = \Phi^4 \delta_{ij} v_i v_j / c'^2$. While the premisses for the present model clearly indicate we are considering a scalar gravitation field, the principle of gravitational affecting of quantities as mass and speed of light give the model a rank-2 tensor-like property.

For the description of photons we rely on the Hamiltonian scheme (or $\lim_{v \rightarrow c}$ of Eq. (50)). In the next Section its accordance with GRT and experiments is shown. The present scalar model thus by-passes the limitation of the ‘classical’ scalar field description in flat space time because essentially it is of the spatially-VSL type.

3 Celestial mechanical GRT-experiments

The ‘basic’ GRT experiments will be described in S_0 perspective; *i*) the deflection of light by the sun, *ii*) the precession of orbital perihelia, *iii*) the gravitational delay of radar echo, and S' perspective; *iv*) the gravitational redshift of spectral lines. The results can be transformed by (\mathbf{x}, t) -GMLT Eqs. (3, 4) and (\mathbf{p}, E) -GMLT Eqs. (21, 22) into the affected S' perspective when necessary.

Hamilton’s principle will be applied from the perspective of the unaffected observer S_0 . Evidently the E -GMLT Eq. (22) between E and E' impedes the simultaneous validity of the principle of energy conservation in both affected and unaffected perspectives in a straightforward manner. For energy conservation relative to S_0 , a static affected observer S' will attribute a change of energy $dE'/E' = -d\Phi/\Phi$. Note however that this asymmetrical feature will resolve the problem of gravitational redshift (see Section 3.4).

The Hamiltonian expression ⁽⁴⁾ Eq. (19) for mass and expression Eq. (20) for light ($m'_0 = 0$) are used:

$$H = \sqrt{m_0^2 c(\mathbf{r})^4 + c(\mathbf{r})^2 p^2} = \Phi(r) \sqrt{m_0'^2 c'^4 + p^2 c'^2 \Phi(r)^2}. \quad (43)$$

For a source with spherical symmetry, Eq. (38), the central force leads to conservation of the angular momentum. The motion is constrained to the (r, φ) plane, with $\theta = \pi/2$, by the Hamilton equations:

$$H = E_0, \quad (44)$$

$$p_\theta = p_{\theta_0} = 0, \quad \dot{\theta} = 0, \quad (45)$$

$$p_\varphi = p_{\varphi_0}, \quad \dot{\varphi} = \frac{c(r)^2}{E_0 r^2} p_{\varphi_0}, \quad (46)$$

$$\dot{p}_r = - \left(E_0 + \left(p_r^2 + \frac{p_{\varphi_0}^2}{r^2} \right) \frac{c(r)^2}{E_0} \right) \frac{\partial_r \Phi(r)}{\Phi(r)} + \frac{c(r)^2}{E_0} \frac{p_{\varphi_0}^2}{r^3}, \quad \dot{r} = \frac{c(r)^2}{E_0} p_r. \quad (47)$$

From which the orbital equation of a test body in a spherical symmetrical field, with conserved quantities $E_0, p_{\theta_0}, p_{\varphi_0}$, is obtained:

$$\frac{dr}{d\varphi} = r \sqrt{\frac{r^2}{c(r)^2} \frac{\left(E_0^2 - m_0^2 c(r)^4 \right)}{p_{\varphi_0}^2} - 1}. \quad (48)$$

The expressions for force \mathbf{f} on, and acceleration \mathbf{g} of, a test mass by a gravitational field are readily obtained as well:

$$\mathbf{f} = -E_0 \left(2 - \frac{1}{\gamma(v)^2} \right) \frac{\nabla \Phi(r)}{\Phi(r)} \quad (49)$$

$$\mathbf{g} = 4\mathbf{v} \frac{\mathbf{v} \cdot \nabla \Phi(r)}{\Phi(r)} - \left(c(\mathbf{r})^2 + v^2 \right) \frac{\nabla \Phi(r)}{\Phi(r)}. \quad (50)$$

3.1 Deflection of light by a gravitational source

The unbound states of light are characterized by a single flexion point ($\dot{p}_r = 0$) in the orbital at the point of minimal approach to the gravitational source. The flexion point and the impact parameter are used to characterize the light orbital. The angle α is defined between the radial vector $\mathbf{1}_r$ and the tangent vector $\mathbf{1}_t$ of the orbital, then: $\tan \alpha = \frac{r d\varphi}{dr}$. The impact parameter b is obtained from asymptotic orbital conditions: $r_{\text{in}} \sin \alpha_{\text{in}} |_{r_{\text{in}} \rightarrow \infty} = b$. Setting the observation of deflection again asymptotically, $r_{\text{out}} \rightarrow \infty$, the deflection angle α_D is defined by:

$$\alpha_D = 2|\varphi_\infty - \varphi_{r_-}| - \pi. \quad (51)$$

The light orbital equation is obtained by putting $m_0 = 0$ in Eq. (48), and using the condition $\dot{p}_{r_-} = 0$ at the point of shortest approach, then $\frac{E_0}{p_{\varphi_0}} = \frac{c'}{b} = \frac{c_-}{r_-}$:

$$\frac{dr}{d\varphi} = r \sqrt{\frac{c_-^2}{c(r)^2} \frac{r^2}{r_-^2} - 1}. \quad (52)$$

⁴The corresponding Lagrangian for particles (39, a) is precisely the one used by Sjödin [23], and the one proposed by Einstein, $-m\sqrt{c^2 - q^2}$, in his early Lorentz-covariant gravitation model ([18], **38**, *Nachtrag zur Korrektur*)

The integration is evaluated in the approximation $O(\kappa^2/r_-^2)$

$$\varphi_\infty - \varphi_r \approx \int_r^\infty \left(1 + 2\kappa \frac{r}{r_-} \frac{1}{r+r_-} \right) \frac{dr}{r\sqrt{r^2 r_-^{-2} - 1}}, \quad (53)$$

$$\approx \arcsin \frac{r_-}{r} + 2\kappa \left(\frac{1}{r_-} - \frac{\sqrt{r^2 - r_-^2}}{r_-(r+r_-)} \right). \quad (54)$$

Then the deflection angle α_D in $O(\kappa^2/r_-^2)$ is given by:

$$\alpha_D = 2 \left| \frac{\pi}{2} + 2\frac{\kappa}{r_-} \right| - \pi. \quad (55)$$

This is the GRT result in coordinate space-time:

$$\alpha_D \approx 4 \frac{GM}{c^2 r_-}, \quad (56)$$

which is the right, observed value.

3.2 The precession of orbital perihelia

The bound state of a massive test body has two flexion points, the perihelion $\frac{dr}{d\varphi}|_{r=r_-} = 0$ and aphelion $\frac{dr}{d\varphi}|_{r=r_+} = 0$ of Eq. (48), in its orbit around a gravitational source. The initial values E_0 and p_{φ_0} can be expressed using the values at the extrema of the bounded orbits:

$$E_0 = m'_0 c'^2 \sqrt{\frac{r_+^2 \Phi_+^{-2} - r_-^2 \Phi_-^{-2}}{r_+^2 \Phi_+^{-4} - r_-^2 \Phi_-^{-4}}}, \quad p_{\varphi_0} = m'_0 c' \sqrt{-\frac{\Phi_+^2 - \Phi_-^2}{\Phi_+^4 r_+^{-2} - \Phi_-^4 r_-^{-2}}}, \quad (57)$$

$$r_+ > r_-, \quad \Phi_+ > \Phi_- : \quad \frac{r_+}{\Phi_+} > \frac{r_-}{\Phi_-}, \quad \frac{r_+^2}{\Phi_+^4} > \frac{r_-^2}{\Phi_-^4}, \quad (58)$$

where the inequalities are obtained directly from $\frac{dr}{d\varphi}|_{r=r_\pm} = 0$. The orbital Eq. (48) can now be expressed using the extremal values:

$$\frac{dr}{d\varphi} = r^2 \sqrt{\frac{1}{\Phi^4} \left(\frac{\Phi_-^4}{r_-^2} \left(\frac{\Phi_+^2 - \Phi_-^2}{\Phi_+^2 - \Phi_-^2} \right) + \frac{\Phi_+^4}{r_+^2} \left(\frac{\Phi_-^2 - \Phi_+^2}{\Phi_-^2 - \Phi_+^2} \right) \right) - \frac{1}{r^2}}. \quad (59)$$

The angular perihelion shift $\Delta\varphi$ is defined by:

$$\Delta\varphi = 2|\varphi(r_+) - \varphi(r_-)| - 2\pi. \quad (60)$$

The integration of the equation requires some standard approximations and substitutions. We approximate till $O(\kappa^3/r^3)$ the integrand according a quadratic form as integrand has zero points at $r = r_-$ and $r = r_+$:

$$\frac{1}{\Phi(r)^4} \left(\frac{f_-}{r_-^2} + \frac{f_+}{r_+^2} \right) - \frac{1}{r^2} \approx -\alpha \left(\frac{1}{r_-} - \frac{1}{r} \right) \left(\frac{1}{r_+} - \frac{1}{r} \right), \quad (61)$$

with α a constant that can be evaluated by considering the value of the integrand for $r \rightarrow \infty$:

$$\alpha = -\frac{r_- r_+}{\Phi_\infty^4} \left(\frac{f_-}{r_-^2} + \frac{f_+}{r_+^2} \right). \quad (62)$$

With;

$$\Phi_\infty = 1, \quad f_{-\infty} = \Phi_-^4 \left(\frac{\Phi_+^2 - 1}{\Phi_+^2 - \Phi_-^2} \right), \quad f_{+\infty} = \Phi_+^4 \left(\frac{\Phi_-^2 - 1}{\Phi_-^2 - \Phi_+^2} \right). \quad (63)$$

Then;

$$\alpha = \frac{r_+ \Phi_-^4}{r_-} \left(\frac{1 - \Phi_+^2}{\Phi_+^2 - \Phi_-^2} \right) - \frac{r_- \Phi_+^4}{r_+} \left(\frac{1 - \Phi_-^2}{\Phi_+^2 - \Phi_-^2} \right). \quad (64)$$

We now consider the integral of the orbital angle:

$$\varphi(r) - \varphi(r_-) \approx \frac{1}{\sqrt{\alpha}} \int_{r_-}^r \frac{dr}{r^2 \sqrt{(r_-^{-1} - r^{-1})(r^{-1} - r_+^{-1})}}. \quad (65)$$

This part of the integration is a standard ([39], sec. 8.6):

$$\varphi(r_+) - \varphi(r_-) \approx -\frac{\pi}{\sqrt{\alpha}}. \quad (66)$$

The perihelion shift angle is then given by:

$$\Delta\varphi \approx 2\pi \left(\frac{1}{\sqrt{\alpha}} - 1 \right). \quad (67)$$

Approximation of $\alpha^{-1/2}$ to order $O(\kappa^2/r^2)$, and $\Phi = \exp[-\kappa/r]$ till $O(\kappa^3/r^3)$ gives:

$$\alpha^{-\frac{1}{2}} = 1 + \frac{3}{2}\kappa \left(\frac{1}{r_+} + \frac{1}{r_-} \right) + O\left(\frac{\kappa^2}{r^2}\right). \quad (68)$$

Then the perihelion shift angle, up to $O(\kappa^2/r^2)$, is given by,

$$\Delta\varphi \approx \frac{6\pi\kappa}{L}, \quad (69)$$

with $L \equiv 1/2(1/r_+ + 1/r_-)$, i.e. the GRT result in coordinate space-time.

It is well known that in reality the found relativistic precession of Mercury is accompanied by a much larger precession of 532'' per century. It is accounted for by the Newtonian interaction of remaining planetary sources (e.g. [39], Section III.9.5). In order to reproduce this effect we should distinguish in the field Eq. (37) the source density $\rho(r^*) = \rho_{Sol.}(r^*) + \rho_{rem.}(r^*)$ with components of the main, Solar, source and the remainder of planetary sources. This leads—in time independent evolution approximation of the remainder planetary sources—to $\Phi(\mathbf{r}) = \exp[\phi(r) + \psi(\mathbf{r})]$, with $\phi(r) = G'M_{Sol.}/(c^2 r)$ and $\psi(\mathbf{r}) = \sum_i G'm_i/(c^2 |\mathbf{r} - \mathbf{r}_i|)$. This addition of effects results, as in GRT ([39], 9.4.11, 9.1.64), to an additional potential term in the gradient term of the acceleration to first approximation:

$$\mathbf{g} = -c^2 \nabla (\phi(r) + 2\phi(r)^2 + \psi(\mathbf{r})) + 4\mathbf{v}\mathbf{v} \cdot \nabla \phi(r) - v^2 \nabla \phi(r). \quad (70)$$

This acceleration expression corresponds only to the fixed field approximation of GRT ([39], 9.2.1), but does account correctly for the precession effect due the remainder Newtonian planetary sources. We refer to [9] for a more realistic—in terms of the true planetary system—time-dependent formulation of source movement in the framework of the present model.

3.3 The gravitational delay of radar echo

The travel time of light is obtained from the variable velocity of light at every point of its orbit between source and observer. From the Hamiltonian equation (47, b), with conserved energy $E_0 = pc$ Eq. (44) and angular momentum p_{φ_0} Eq. (46) we obtain:

$$\dot{r} = c(r) \sqrt{1 - \frac{p_{\varphi_0}^2}{E_0^2} \frac{c(r)^2}{r^2}}. \quad (71)$$

The radial velocity vanishes at the point of closest approach near the Sun, $r = r_-$, i.e. $\dot{r}|_{r=r_-} = 0$ giving $p_{\varphi_0}/E_0 = r_-/c_-$. Then, light traveling over its orbit from r_i to r_f takes the time Δt :

$$\Delta t = \int_{r_i}^{r_f} \frac{dr}{c(r) \sqrt{1 - r_-^2 r^{-2} c_-^{-2} c(r)^2}}. \quad (72)$$

The travel time $\Delta t(r, r_-)$, approximated till $O\left(\frac{\kappa^2}{r^2}\right)$, is found to be:

$$\Delta t(r, r_-) \approx \frac{1}{c'} \int_{r_-}^r \left(1 + 2\frac{\kappa}{r} + \frac{\kappa r_-}{r(r + r_-)}\right) \frac{1}{\sqrt{1 - r_-^2 r^{-2}}} dr. \quad (73)$$

The three components of the integrand lead respectively to:

$$\Delta t(r, r_-) \approx \frac{\sqrt{r^2 - r_-^2}}{c'} + \frac{\kappa}{c'} \left(2 \ln \frac{r + \sqrt{r^2 - r_-^2}}{r_-} + \left(\frac{r - r_-}{r + r_-}\right)^{\frac{1}{2}}\right). \quad (74)$$

i.e. the delay term of GRT in coordinate time ([39], equation 8.7.4).

3.4 Gravitational shift of spectral lines

The effect of gravitation on the frequency of light over its orbit is calculated using the Hamiltonian description Eq. (43) and the Einstein-Compton relations for corpuscular light Eq. (28). In the unaffected perspective of S_0 the conservation of energy $E = E_0$, Eq. (44), is satisfied on the light orbital. Accordingly the unaffected observers S_0 attributes to light the frequency and wavelength:

$$\dot{E} = 0 \rightarrow h\dot{\nu} = 0, \quad \nu = \nu_0, \quad (75)$$

$$\frac{\dot{\lambda}}{\lambda} = \frac{\dot{c}}{c}, \quad \lambda = \lambda_0 \frac{\Phi^2}{\Phi_0^2}. \quad (76)$$

The constancy of the light frequency relative to S_0 is the trivial result of energy conservation. The variation of wave length of light relative to S_0 is proportional to Φ^2 . Therefore S_0 will attribute a ‘wave-length spectral shift’: e.g. for light emitted by a source at r_s and observed at r_o , $r_o \gg r_s$:

$$\frac{\lambda_s - \lambda_o}{\lambda_o} = -2\frac{\kappa}{r_s} + O\left(\frac{\kappa}{r_o}, \frac{\kappa^2}{r_s^2}\right) \quad (77)$$

Therefore S_0 observes not a shift of frequency, but a ‘red shift of the wavelength’ twice the magnitude an affected observer is expected to measure.

In the present scalar gravitation model the correct gravitational spectral shift of light is restored in the perspective of affected observers S' only. With the GMLT-invariance of Planck’s constant, Eq. (30), the observations of S_0 can be transformed into affected observations of S' by either

(\mathbf{x}, t) -GMLT Eqs. (3, 4) or (\mathbf{p}, E) -GMLT Eqs. (21, 22). Both GMLT's lead, for $\mathbf{u}' = 0$, to the S_0 to S' relation for frequency and wave length:

$$\nu' = \frac{\nu}{\Phi(r)}, \quad (78)$$

$$\lambda' = \frac{\lambda}{\Phi(r)}. \quad (79)$$

The affected observer S' thus attributes to light, traveling from a source at r_s to position r_o with $r_o \gg r_s$, a frequency and wave length shift equal to:

$$\frac{\nu'_s - \nu'_o}{\nu'_o} = \frac{\Phi(r_o)}{\Phi(r_s)} - 1 = \frac{\kappa}{r_s} + O\left(\frac{\kappa}{r_o}, \frac{\kappa^2}{r_s^2}\right), \quad (80)$$

$$\frac{\lambda'_s - \lambda'_o}{\lambda'_o} = \frac{\Phi(r_s)}{\Phi(r_o)} - 1 = -\frac{\kappa}{r_s} + O\left(\frac{\kappa}{r_o}, \frac{\kappa^2}{r_s^2}\right). \quad (81)$$

The gravitational spectral red shift as predicted by GRT is recovered by the affected observer S' . The correct frequency and wave length spectral shift relative to a static S' , as compared to S_0 , is of course due to the nonconservation of energy relative to S' .

4 Conclusions

We presented the foundations of a scalar Lorentz-covariant gravity model based on alternative metrization by consistent scaling of physical quantities in line with Poincaré's geometric conventionalism and Lorentz-Poincaré type interpretation. Two levels of description are related, the gravitationally affected observer (curved metric) and gravitationally unaffected observer (flat metric), by appropriate gravitationally modified Lorentz transformations or GMLT's. The inherent gravitational scaling function is nonsingular at any finite distance to a point source, while it is mathematically characterized by the Schwarzschild radius.

The model assumes an intrinsic spatially variable speed of light, depending on the gravitational field, but the GMLT's assure its observed local invariance. The presence of the variable speed of light, in the term v^2/c^2 in the Lagrangian, lends the model rank-2 tensorial properties. In this sense our 'modified' scalar model by-passes the critique on 'standard' scalar models in flat space [26]. The Hamiltonian for a test particle in a gravitational field obtained by energy-momentum GMLT expresses an essential effect according Mach's mass induction principle. Developed for particles and photons in a static spherically symmetric gravitation field, it reproduces till required Post-Newtonian order the results for the four 'basic' experiments of General Relativity Theory.

The here presented work focused the simpler static spherically symmetric configuration with test masses only and thus ignored effects, e.g. due to: size and proper kinematics of sources and gravitating objects, or gravitational waves. Some of the latter extensions of configuration will be covered in upcoming work concerning this scalar model [9, 10].

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A Appendix

Space and Time GMLT between affected observers

The space time GMLT of the second type, *i.e.* between two distinctly, gravitationally and kinematically, affected observers S'_1 and S'_2 , $\Phi_1 \neq \Phi_2$, $\mathbf{u}_1 \not\parallel \mathbf{u}_2$ is obtained by elimination of the S_0 perspective from a composition of two GMLT's (5, 6) and (3, 4). This relation should exclude specific kinematic reference to the perspective of S_0 in order not to obtain referenceless kinematical expressions in the homogenous case $\Phi_1 = \Phi_2$. The second type GMLT, exclusively in terms of kinematical quantities relative to S'_1 and S'_2 , are given by :

$$d\mathbf{x}'_1 = d\mathbf{x}'_{2\perp\{12,21\}} \frac{\Phi_2}{\Phi_1} + \Gamma_{21} \frac{\Phi_1}{\Phi_2} \mathbf{u}'_{21} dt'_2 - \Gamma_{21} \frac{\Phi_1}{\Phi_2} \frac{d\mathbf{x}'_2 \cdot \mathbf{u}'_{12}}{u'^2_{12}} \mathbf{u}'_{21} +$$

$$\frac{d\mathbf{x}'_2 \cdot \mathbf{u}'_{21\perp 12}}{u'^2_{21\perp 12}} \frac{\Phi_2}{\Phi_1} \left[\mathbf{u}'_{21} \left(1 + A \frac{\Phi_1^4}{\Phi_2^4} \left(\frac{1}{\Gamma_{12}} - 1 \right) - B \frac{\Phi_1^2}{\Phi_2^2} \Gamma_{21} \left(1 + \frac{\gamma_1}{\Gamma_{12}} \right) \right) \right.$$

$$\left. + \mathbf{u}'_{12} \left(\cos T \left| \frac{u'_{21}}{u'_{12}} \right| + A \frac{\Phi_1^2}{\Phi_2^2} \left(1 - \frac{1}{\Gamma_{21}} \right) - B (1 + \gamma_2) \right) \right], \quad (82)$$

$$dt'_1 = \left(dt'_2 - \frac{d\mathbf{x}'_2 \cdot \mathbf{w}'_{12}}{c'^2} \right) \Gamma_{21} \frac{\Phi_1}{\Phi_2}, \quad (83)$$

with:

$$A = \frac{u'^2_{21\perp 12}}{c'^2} \frac{(1 + \gamma_1)(1 + \gamma_2)}{\Gamma_{12} D^2}, \quad B = \frac{\mathbf{u}'_{21\perp 12} \cdot \mathbf{w}'_{12}}{c'^2} \frac{1}{D}, \quad (84)$$

$$D = -\frac{(1 + \gamma_1)(1 + \gamma_2)}{\Gamma_{12} \Gamma_{21}} + \left(1 + \frac{\gamma_1}{\Gamma_{12}} \right) \left(1 + \frac{\gamma_2}{\Gamma_{21}} \right), \quad (85)$$

where the gamma factors γ_1 and γ_2 are expressed in terms relative to S'_1 and S'_2 according:

$$\gamma_1 = \sqrt{\frac{\Gamma_{12}^2 \gamma_{12}^{-2} - \Phi_1^4 \Phi_2^{-4}}{1 - \Phi_1^4 \Phi_2^{-4}}}, \quad \gamma_2 = \sqrt{\frac{\Gamma_{21}^2 \gamma_{21}^{-2} - \Phi_2^4 \Phi_1^{-4}}{1 - \Phi_2^4 \Phi_1^{-4}}}, \quad (86)$$

and in which the ‘gamma’ factors for kinematical contraction and dilation are:

$$\Gamma_{21} \equiv \frac{\Phi_2}{\Phi_1} \frac{dt'_1}{dt'_2} \Big|_{d\mathbf{x}'_2=0} = \gamma_1 \gamma_2 \left(1 - \frac{\mathbf{u}'_1 \cdot \mathbf{u}'_2}{c'^2} \frac{\Phi_2^2}{\Phi_1^2} \right), \quad (87)$$

$$\Gamma_{12} \equiv \frac{\Phi_1}{\Phi_2} \frac{dt'_2}{dt'_1} \Big|_{d\mathbf{x}'_1=0} = \gamma_1 \gamma_2 \left(1 - \frac{\mathbf{u}'_1 \cdot \mathbf{u}'_2}{c'^2} \frac{\Phi_1^2}{\Phi_2^2} \right), \quad (88)$$

and where the relative frame velocities are:

$$\mathbf{u}'_{12} \equiv \frac{d\mathbf{x}'_2}{dt'_2} \Big|_{d\mathbf{x}'_1=0} = \frac{\mathbf{u}'_2 - (\mathbf{u}'_{1\parallel 2} + \mathbf{u}'_{1\perp 2} \gamma_2^{-1}) \Phi_1^2 \Phi_2^{-2}}{(1 - \mathbf{u}'_1 \cdot \mathbf{u}'_2 c'^{-2} \Phi_1^2 \Phi_2^{-2})}, \quad (89)$$

$$\mathbf{u}'_{21} \equiv \frac{d\mathbf{x}'_1}{dt'_1} \Big|_{d\mathbf{x}'_2=0} = \frac{\mathbf{u}'_1 - (\mathbf{u}'_{2\parallel 1} + \mathbf{u}'_{2\perp 1} \gamma_1^{-1}) \Phi_2^2 \Phi_1^{-2}}{(1 - \mathbf{u}'_1 \cdot \mathbf{u}'_2 c'^{-2} \Phi_2^2 \Phi_1^{-2})}, \quad (90)$$

and the ‘counter-scaled’ relative frame velocities are:

$$\mathbf{w}'_{12} \equiv \frac{\mathbf{u}'_2 - (\mathbf{u}'_{1\parallel 2} + \mathbf{u}'_{1\perp 2} \gamma_2^{-1}) \Phi_2^2 \Phi_1^{-2}}{1 - \mathbf{u}'_1 \cdot \mathbf{u}'_2 c'^{-2} \Phi_2^2 \Phi_1^{-2}}, \quad \mathbf{w}'_{21} \equiv \frac{\mathbf{u}'_1 - (\mathbf{u}'_{2\parallel 1} + \mathbf{u}'_{2\perp 1} \gamma_1^{-1}) \Phi_1^2 \Phi_2^{-2}}{(1 - \mathbf{u}'_1 \cdot \mathbf{u}'_2 c'^{-2} \Phi_1^2 \Phi_2^{-2})}, \quad (91)$$

and where the ‘Thomas angle’ is given by:

$$\cos T = -\frac{\mathbf{u}'_{12} \cdot \mathbf{u}'_{21}}{u'_{12} u'_{21}}. \quad (92)$$

The precise mathematical group symmetry of the modified Lorentz transformations (82, 83) is not apparent. Only the space intervals orthogonal to the kinematical plane $\{\mathbf{u}'_{12}, \mathbf{u}'_{21}\}$ follow a simple scaling transformation. In the kinematical plane the analysis of the transformations into a boost and Thomas rotation is not recovered, which is clear from the difference in amplitude of u'_{12} and u'_{21} , Eqs. (89, 90).

Only in the homogeneous case, when the Lorentz symmetry is recovered due $\Phi_1 = \Phi_2$ and $\Gamma_{21} = \Gamma_{12} = \Gamma$, do the coefficients in the transformation correspond to trigonometric functions of the Thomas angle T :

$$\cos T|_{\Phi_1=\Phi_2} = \frac{(1 + \Gamma + \gamma_1 + \gamma_2)^2}{(1 + \Gamma)(1 + \gamma_1)(1 + \gamma_2)} - 1. \quad (93)$$

In the general case the kinematical affecting factors do not correspond to the standard SRT form of $\gamma_{21} \equiv (1 - u'^2_{21}/c^2)^{-1/2}$ and $\gamma_{12} \equiv (1 - u'^2_{12}/c^2)^{-1/2}$. Their general relation is:

$$\gamma_{21} = \Gamma_{21} \frac{\gamma_2^*}{\gamma_2}, \quad \gamma_2^* = (1 - u'^2_2 c'^{-2} \Phi_2^4 \Phi_1^{-4})^{-1/2}, \quad (94)$$

$$\gamma_{12} = \Gamma_{12} \frac{\gamma_1^*}{\gamma_1}, \quad \gamma_1^* = \left(1 - u'^2_1 c'^{-2} \Phi_1^4 \Phi_2^{-4}\right)^{-1/2}. \quad (95)$$

Some modified relations of SRT are valid in the general inhomogeneous case:

$$\mathbf{w}'_{21} \cdot \mathbf{u}'_{21} = \mathbf{w}'_{12} \cdot \mathbf{u}'_{12}, \quad (96)$$

$$\frac{1}{\Gamma_{12}\Gamma_{21}} = 1 - \frac{\mathbf{w}'_{12} \cdot \mathbf{u}'_{12}}{c'^2}. \quad (97)$$

The frame velocities of S'_1 and S'_2 relative to S_0 , or the gravitational field Φ , can be related to quantities relative to S'_1 and S'_2 by inverting Eqs. (89, 90):

$$\mathbf{u}'_2 = \left(\mathbf{u}'_{21} \frac{\gamma_1}{\Gamma_{12}} \frac{\Phi_1^2}{\Phi_2^2} + \mathbf{u}'_{12} \frac{1 + \gamma_2/\Gamma_{21}}{1 + 1/\gamma_1} \right) \frac{1}{D} \left(1 + \frac{1}{\gamma_1} \right) \left(1 + \frac{1}{\gamma_2} \right), \quad (98)$$

$$\mathbf{u}'_1 = \left(\mathbf{u}'_{12} \frac{\gamma_2}{\Gamma_{21}} \frac{\Phi_2^2}{\Phi_1^2} + \mathbf{u}'_{21} \frac{1 + \gamma_1/\Gamma_{12}}{1 + 1/\gamma_2} \right) \frac{1}{D} \left(1 + \frac{1}{\gamma_1} \right) \left(1 + \frac{1}{\gamma_2} \right). \quad (99)$$

Energy and Momentum GMLT between affected observers

Due the similar kinematic structure and the inverse overall affecting factor of (\mathbf{p}, E) -GMLT (23, 24) and (\mathbf{x}, t) -GMLT (5, 6), the general energy-momentum transformation between affected observers can be deduced from their space-time relation (if $\Phi_i \rightarrow 1/\Phi_i$, then $\Gamma_{21} \rightarrow \Gamma_{12}$, $\mathbf{u}'_{12} \rightarrow \mathbf{w}'_{12}$, ... while γ_i remain invariant). The most general energy-momentum GMLT between two distinctly affected observers S'_1 and S'_2 , $\Phi_1 \neq \Phi_2$, $\mathbf{u}_1 \not\parallel \mathbf{u}_2$ is then given by:

$$\begin{aligned} \mathbf{p}'_1 &= \mathbf{p}'_{2\perp\{12,21\}} \frac{\Phi_1}{\Phi_2} + \Gamma_{12} \frac{\Phi_2}{\Phi_1} \mathbf{w}'_{21} \frac{E'_2}{c'^2} - \Gamma_{12} \frac{\Phi_2}{\Phi_1} \frac{\mathbf{p}'_2 \cdot \mathbf{w}'_{12}}{w'^2_{12}} \mathbf{w}'_{21} + \\ &\frac{\mathbf{p}'_2 \cdot \mathbf{w}'_{21\perp 12}}{w'^2_{21\perp 12}} \frac{\Phi_1}{\Phi_2} \left[\mathbf{w}'_{21} \left(1 + \tilde{A} \frac{\Phi_2^4}{\Phi_1^4} \left(\frac{1}{\Gamma_{21}} - 1 \right) - \tilde{B} \frac{\Phi_2^2}{\Phi_1^2} \Gamma_{12} \left(1 + \frac{\gamma_1}{\Gamma_{21}} \right) \right) \right. \\ &\quad \left. + \mathbf{w}'_{12} \left(\cos \tilde{T} \left| \frac{w'_{21}}{w'_{12}} \right| + \tilde{A} \frac{\Phi_2^2}{\Phi_1^2} \left(1 - \frac{1}{\Gamma_{12}} \right) - \tilde{B} (1 + \gamma_2) \right) \right], \end{aligned} \quad (100)$$

$$E'_1 = (E'_2 - \mathbf{p}'_2 \cdot \mathbf{u}'_{12}) \Gamma_{12} \frac{\Phi_2}{\Phi_1}, \quad (101)$$

where $\mathbf{w}'_{21\perp 12}$ now indicates the part of \mathbf{w}'_{21} orthogonal and coplanar with \mathbf{w}'_{12} , and:

$$\tilde{A} = \frac{w'_{21\perp 12}{}^2}{c'^2} \frac{(1+\gamma_1)(1+\gamma_2)}{\Gamma_{21}\tilde{D}^2}, \quad \tilde{B} = \frac{\mathbf{w}'_{21\perp 12} \cdot \mathbf{u}'_{12}}{c'^2} \frac{1}{\tilde{D}}, \quad (102)$$

$$\tilde{D} = -\frac{(1+\gamma_1)(1+\gamma_2)}{\Gamma_{12}\Gamma_{21}} + \left(1 + \frac{\gamma_1}{\Gamma_{21}}\right) \left(1 + \frac{\gamma_2}{\Gamma_{12}}\right), \quad (103)$$

$$\cos \tilde{T} = -\frac{\mathbf{w}'_{12} \cdot \mathbf{w}'_{21}}{w'_{12}w'_{21}}. \quad (104)$$

Let us for example consider the configuration with S'_2 kinematically coincident with the test body, *i.e.* $\mathbf{p}'_2 = 0$. Then energy and momentum attributed by S'_1 relate to the quantities in the self frame S'_2 , with $E'_2 = m'_0 c'^2$, according:

$$\mathbf{p}'_1 = \Gamma_{12} \frac{\Phi_2}{\Phi_1} m'_0 \mathbf{w}'_{21}, \quad (105)$$

$$E'_1 = \Gamma_{12} \frac{\Phi_2}{\Phi_1} m'_0 c'^2. \quad (106)$$

While the velocity of S'_2 relative to S'_1 is \mathbf{u}'_{21} , we obtain an expression for momentum \mathbf{p}'_1 along the counter-scaled velocity \mathbf{w}'_{21} . Thus dismissing, in this model, the conventional momentum definition in the gravitationally affected perspective.

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